M.Sc. (Part—I) Semester—I (C.B.C.S. Scheme) Examination

STATISTICS

(Estimation Theory)

Paper—II

Time : Three Hours] [Maximum Marks : 80

N.B. :— Solve either (A) or (B) part from each question.

1. (A) (i) Define unbiased estimator and show that unbiased estimator need not be unique. 8

(ii) Define consistency. If $X \sim N(\mu, \sigma^2)$ then show that sample mean $\overline{x}$ is consistent estimator of μ. 8

OR

(B) (i) Explain the following terms :

(a) Likelihood function

(b) Likelihood equivalence. 6

(ii) Define MVUE. Show that it is unique. 10

2. (A) (i) Obtain MLE of μ on the basis of r.s. of size n from $N(\mu, \sigma^2)$:

(a) When σ^2 is known

(b) When σ^2 is unknown. 10

(ii) State only Cramer Huzurbazar theorem along with all regularity conditions. 6

OR

(B) (i) Describe method of moment with example. 8

(ii) Explain the method of maximum likelihood and show that MLE need not be unbiased. 8

3. (A) (i) State and prove Cramer Rao inequality with its regularity conditions. 10

(ii) Explain single parameter exponential family. Let $X \sim N(\mu, \sigma^2)$ where μ is known and σ^2 is unknown, then show that family belongs to one parameter exponential family. 6

OR

(B) (i) Define sufficiency. State and prove factorization theorem. Also show that if r.s. are from normal population with mean θ_1 and variance θ_2, then obtain sufficient statistic for θ_1 and θ_2. 10

(ii) Explain with example the Pitman family of distribution. 6

4. (A) (i) State and prove Rao Blackwell theorem. 10

(ii) Define completeness. Prove that Poisson family is a complete family. 6

OR

(B) (i) Explain the meaning of :

(a) Bounded completeness

(b) Blackwellization

(c) Minimal sufficient complete statistic. 8

(ii) State and prove Lehmann-Scheffe theorem. 8

5. (A) (i) Explain the terms :

(a) Confidence interval

(b) Degree of confidence interval

(c) Shortest length C.I. 8

(ii) Construct $(1-\alpha) \times 100\%$ confidence interval for mean of normal distribution when σ^2 is known. 8

OR

(B) (i) Construct $(1-\alpha)100\%$ confidence interval for σ^2 on the basis of r.s. from $N(\mu, \sigma^2)$ when μ is known. 8

(ii) Define :

(a) Lower and upper confidence bound

(b) Pivot

(c) Confidence coefficient. 8