(ii) Define Poisson process. Stating the underlying postulates clearly, derive the difference differential equation for it. 8+8

5. (A) Define Wiener process. Explain that Wiener process is a limiting case of random walk. 16

OR

(B) Derive the forward difference equation to the Wiener process. 16

AQ – 833

First Semester M.A./M.Sc. (Part – I) Examination

STATISTICS

Paper – III

1SCA3 : Stochastic Processes

P. Pages : 4

Time : Three Hours] [Max. Marks : 80

Note : Solve either A or B from each question.

1. (A) (a) Define stochastic process. Give classification of stochastic process, on the basis of nature of state space and parameter space with an example of each.

(b) Define transition probability and transition probability matrix. Also explain stationarity of stochastic process. 8+8

OR

(B) (i) Consider the stochastic process $x(t)=A_1+A_2t$ where $E(Ai)=ai$, $V(Ai)=\sigma i^2$, $i=1,2$, A_1 and A_2 are independent then show that $cov\ (x(t), x(s)) = \sigma_1^2 + t.s.\sigma_2^2$

(ii) Define Markov chain. The transition probability matrix of a Markov chain having three states 1, 2 and 3 is

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

and initial distribution is $\pi_0 = (0.7, 0.2, 0.1)$, find $P\{x_3=2,\ x_2=3,\ x_1=3,\ x_0=2\}$

8+8

2. (A) (a) State and prove Chapman-Kolmogorov equation.

(b) If state j of a Markov chain is persistent then show that

$$\sum_{n=0}^{\infty} p_{jj}^{n} = \infty$$ 8+8

OR

(B) (i) If state j is persistent then show that for every state k that can be reached from state j $F_{kj} = 1$

(ii) For two state Markov chain with tpm

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, \quad 0<a,\ b<1$$

Show that

$$P^n = \frac{1}{a+b}\begin{bmatrix} b & a \\ b & a \end{bmatrix} + \frac{(1-a-b)^n}{a+b}\begin{bmatrix} a & -a \\ -b & b \end{bmatrix}$$

8+8

3. (A) Derive the expression for Gambler's ruin when $p = q$. 16

OR

(B) Derive the expression for expected duration of the one dimentional random walk game when $p \neq q$. 16

4. (A) (a) If N(t) is a Poisson process and $s<t$ then show that

$$P\{N(s)=k / N(t)=n\} = {}^{n}c_{k} \cdot (s/t)^k (1-s/t)^{n-k}$$

(b) Derive the difference differential equation for Birth and Death Process.

8+8

OR

(B) (i) If N(t) is a poisson process then prove that the autocorrelation coefficient between N(t) and N(t+s) is $\left[\frac{t}{t+s}\right]^{1/2}$