AU–248

M.A./M.Sc. (Part—I) Semester—I (C.B.C.S. Scheme) Examination

STATISTICS

(Stochastic Processes)

Paper—III

Time : Three Hours] [Maximum Marks : 80

Note :— Solve either A or B from each question.

1. (A) (i) Define Stochastic Process. Explain following type of Stochastic process with example :
   (a) Discrete in state space and discrete in time
   (b) Continuous in state space and continuous in time.

   (ii) Obtain the joint probability distribution of $x_0, x_1, x_2, ..., x_n$ in terms of transition probabilities P_{jk} and the initial distribution of x_0 for a Markov Chain $\{x_n, n \geq 0\}$.

   8+8

OR

   (B) (i) Define :
   (a) Markov process
   (b) Stationary process
   (c) Transition probability.

   (ii) Define a Stochastic process. Give one example of a Stochastic process and give its classification according to state space and time domain. 8+8

2. (A) (i) Define Recurrent state, Transient state and Communicating state of a Markov Chain.

   (ii) Show that a finite Markov chain cannot have all transient states.

   (iii) State and prove Chapman-Kolmogorov equation. 4+4+8

OR

   (B) (i) State and prove ergodic theorem.

   (ii) State applications of Stochastic process in Biological and Physical Sciences.

   (iii) Show that in an irreducible Markov chin every state is of same type. 8+4+4

3. (A) (i) State Gambler's Ruin problem and obtain the probability of ultimate ruin of the gambler.

   (ii) Discuss two dimensional random walk. 8+8

OR

(B) (i) Define a Markov chain.

A particle performs a random walk with absorbing barrier say at 0 and 4 whenever it is at any position 'r' $(0 < r < 4)$ it moves to $r + 1$ with probability 'p' and to $r - 1$ with probability q such that $p + q = 1$. Write the transition probability matrix of Markov chain.

(ii) Explain a three dimensional random walk. Show that the state represented by origin is a transient state. 8+8

4. (A) (i) Describe pure birth process and obtain the difference differential equation of pure birth process.

(ii) State the properties of Poisson process and prove any one. 8+8

OR

(B) (i) Derive the difference differential equation of pure birth process.

(ii) If X(t) is a Poisson process with parameter λ, derive mean and variance of X(t). 8+8

5. (A) (i) Let $\{X(t), t > 0\}$ be a Wiener process with drift μ and variance σ^2. Let $X(0) = X_0$ and let $a > 0$ be an absorbing barrier. Find the probability of eventual absorption at a.

(ii) Discuss continuous state space continuous time Markov chain. 10+6

OR

(B) Define Wiener process. Show that Wiener process can be obtained as limit of random walk. 16