AV–1897

M.Sc. (Part—I) Semester—I (C.B.C.S. Scheme) Examination

STATISTICS

(Stochastics Processes)

Paper—III

Time : Three Hours] [Maximum Marks : 80

N.B. :— Solve either (A) or (B) part from each question.

1. (A) (i) Write the classification of Stochastic Process according to state space and time domain with example.

(ii) Define :

(a) Gaussian process

(b) Transition probability

(c) n-step transition probability

(d) Markov chain. 8+8

OR

(B) (i) Define :

(a) Markov process

(b) Stationary process

(c) Order of Markov chain

(d) Steady state probability.

(ii) Let $P = \begin{bmatrix} \alpha & 1-\alpha \\ 1-\beta & \beta \end{bmatrix}$ be a transition probability matrix of a Markov chain $0 < \alpha, \beta < 1$. Derive $\lim\limits_{n \to \infty} P^n$. 8+8

2. (A) (i) State and prove Chapman-Kolmogorov equation.

(ii) Let $f_{jk}^{(n)}$ denote probability that the system starting from state 'j' reaches to state 'k' for the first time in n jumps and p_{jk}^{n} denote the probability that the system reaches state k after n jumps obtain relation between p_{jk}^{n} and $f_{jk}^{(n)}$. 8+8

OR

(B) (i) Define :

(a) Recurrent state

(b) Transient state

(c) Periodic state

(d) Ergodic state.

(ii) Define persistent state. Prove that the state j is persistent or transient according as $\sum\limits_{n=0}^{\infty} p_{jj}^{(n)} = \infty$ or $< \infty$. 8+8

3. (A) (i) Obtain the expected duration of a game if the gambler starts with initial capital Z and the total capital of two players is 'a' in the classical gambler's ruin problem.

(ii) Define random walk and gambler's ruin problem. What is the similarity between the two ? Show that it is a particular Markov chain. Obtain transition probability if barriers are absorbing. 8+8

OR

(B) (i) Define :

(a) Absorbing barrier

(b) Reflecting barrier

(c) Elastic barrier with example.

(ii) Explain in brief two and three dimensional random walk. 8+8

4. (A) (i) State postulates of Poisson process. Obtain an expression for $P[X(t) = n] = P_n(t)$ (in usual notation). n = 0, 1, 2, ...

(ii) Show that mean rate i.e. parameter λ of the Poisson process $\{X(t)\}$ can be reasonably estimated by the observation $\frac{X(t)}{t}$ for large t. 8+8

OR

(B) (i) If X(t) is Poisson process with parameter λ derive mean and variance of X(t).

(ii) Describe birth death process. Obtain the difference differential equation of birth death process. 8+8

5. (A) Define Weiner process and show that it can be regarded as a limit of random walk. 16

OR

(B) (i) Discuss continuous time continuous state space Markov process with example.

(ii) Discuss first passage problem in case of Wiener process. 6+10