AQ–880

M.Sc. Part—I (Semester—II) (CBCS Scheme) Examination (Old)

MATHEMATICS

(Advanced Discrete Mathematics—II)

Paper—2 MTH 5 (Optional)

Time—Three Hours] [Maximum Marks—80

Note :— Solve **ONE** question from each Unit.

UNIT—I

1. (a) Define Euler graph and prove that connected graph G is an Euler graph. 8

   (b) A simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges, prove this. 8

2. (c) Prove that the number of vertices of odd degree in a graph is always even. Also define degree of vertex. 8

   (d) Prove that in a simple diagraph G = <V, E>, every node of the diagraph lies in exactly one strong component. 8

UNIT—II

3. (a) Prove that with respect to a given spanning tree T, a chord Ci that determines a fundamental circuit Γ occurs in every fundamental cutset associated with the branches in Γ and in no other. 8

   (b) Prove that a connected graph G is an Euler graph if degree of every vertex of G is even. 8

4. (c) Show that two graphs G_1 and G_2 are isomorphic iff their matrices $A(G_1)$ and $A(G_2)$ differ only be permutation of rows and columns. 8

   (d) If A(G) is an incidence matrix of a connected graph G with n vertices then prove that rank of A(G) is $n - 1$. 8

UNIT—III

5. (a) Let s be any finite state machine & x & y be any words then show that :

$$\delta(s; xy) = \delta(\delta(s, x), y)$$

and $\lambda(s; xy) = \lambda(\delta(s, x), y)$. 8

   (b) Prove that :

   (i) If $M_1 = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ is a Moore machine then there is a Melay machine M_2 equivalent to M_1.

   (ii) If $M_1 = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ is a Melay machine then there is a Moore machine M_2 equivalent to M_1. 8

6. (c) Let A = {a, b}, construct an automaton M which will accept precisely those words from A which end in two b's. Prove this. 8

   (d) Prove that if for some integer k, $P_{k+1} = P_k$ then $P_k = P$ and conversely. 8

UNIT—IV

7. (a) Define Sentential form. Give two examples fo grammar. 8

   (b) Prove that the class of regular set is closed under quotient with arbitary sets. 8

8. Show that $L = \{a^b; b \text{ is prime}\}$ is not a regular set. 16

UNIT—V

9. (a) Write the rules of identifier, describe with an example. 8

   (b) Explain the reverse polish notations with illustrations. 8

10. (c) Explain Turing machine with its recognition by an illustration. 8

   (d) Prove that the rank of any well formed polish formula is 1 and the rank of any proper head of a polish formula is greater than or equal to 1. Conversely if the ranks are as described earlier then prove that the formula is well-formed. 8