M.Sc. (Part—I) Semester—II (CBCS Scheme) Examination
MATHEMATICS
(Advanced Discrete Maths—II)
Paper—206 (Optional)

Time : Three Hours] [Maximum Marks : 80

Note :— Solve **ONE** question from each unit.

UNIT—I

1. (a) Show that a simple diagraph is strongly connected iff there is a cycle in G which includes each node at least once and no isolated node. 8

   (b) Define planar graph and show that every complete graph with five vertices is non planar. 8

2. (c) Prove the following statements are equivalent :

   (i) There is exactly one path between every pairs of vertices in G.

   (ii) G is minimally connected graph. 8

   (d) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 8

UNIT—II

3. (a) If A(G) is an incidence matrix of a connected graph with n-vertices then prove that rank of A(G) is $n - 1$. 8

   (b) Prove that every connected graph has at least one spanning tree. 8

4. (c) Prove that with respect to any of its spanning trees, a connected graph of n vertices and e edges has $n - 1$ tree branches and $e - n + 1$ chords. 8

   (d) Prove that the rank of cut-set matrix C(G) is equal to the rank of the incidence matrix A(G), which equals the ranks of graph G. 8

UNIT—III

5. (a) Define Moore machine and Mealy machine. Prove that if $M_1 = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ is a Mealy machine then there is a Moore machine M_2 equivalent to M_1. 8

   (b) Prove that if $P_K \neq P$, then the cardinality of P is greater than or equal to $K + 1$. 8

6. (c) Let $G(V_N, V_T, S, \phi)$ be T_3 grammar which generates the language L(G). Then prove that there exists a finite-state acceptor $M = (V_T, Q, S, \delta, F)$ s.t. $T(M) = L(G)$. 8

   (d) Let S be any finite state machine and x and y be any words then show that :

   $\delta(s; xy) = \delta(\delta(s, x), y)$ and $\lambda(s; xy) = \lambda(\delta(s, x), y)$. 8

UNIT—IV

7. (a) Explain context free and context sensitive grammars. 8

   (b) Prove that $S_1 = S_2$ iff the substring S_1S_2 appears in a handle of some sentential form. 8

8. (c) Construct a grammar for the language $L = \{x | x \in \{a,b\}^*$, the number of a's in x is a multiple of 3$\}$. 8

   (d) Explain context free and context sensitive grammars. 8

UNIT—V

9. (a) Write note on conversion of infix expressions to polish notations. 8

   (b) Explain the reverse polish notation with illustrations. 8

10. (c) Translate Infix string (a + b ^ c ^ d) ≠ (e + f d) to polish. 8

    (d) Explain Turing machine with its recognition by an illustration. 8