8. (c) Using the method of successive approximations,

solve : $y(x) = 1 + \int_0^x (x - t)\, y(t)\, dt$ taking $y_0(x) = 0$.

8

(d) Prove that if the kernel is symmetric then all its iterated kernels are also symmetric. 8

UNIT—V

9. Define Green's function. Solve the boundary value problem,

$$y'' + \pi^2 y = \cos \pi x,\ y(0) = y(1),\ y'(0) = y'(1)$$

using Green's functions. 2+14

10. Solve $y'' + y = x^2$; $y(0) = y(\pi/2) = 0$ using Green's function. 16

M.Sc. Part—I (Semester—II) (CBCS Scheme) Examination (New)

MATHEMATICS

(Integral Equation)

Paper—VIII (203)

Time—Three Hours] [Maximum Marks—80

Note :— Solve ONE question from each Unit.

UNIT—I

1. (a) Define Homogeneous Volterra integral equation of second kind. Also verify whether $y(x) = 2e^x(x - 1/3)$ is the solution of

$$y(x) = 2xe^x - \int_0^1 e^{x-t} y(t)\, dt.$$

2+6

(b) Reduce the following boundary value problem into an integral equation :

$$\frac{d^2y}{dx^2} + \lambda y = 0 \text{ with } y(0) = 0 \text{ and } y(\ell) = 0.$$

8

2. (c) Define Linear i g·al equation. Also show that $y(x) = 1 - x$ is : solution of integral equation,

$$\int_0^x e^{x-t}\, y(t)\, dt =$$ 2+6

(d) Transform $\frac{d^2y}{dx^2}$ · ·· = 1 with $y(0) = 0$ and $y(1) = 1$ into an int· ·l equation. 8

UN·· –II

3. (a) Determine · ·igen values and eigen functions of the homoge· ·s integral equation

$$y(x) = \lambda \int_0^{\pi} \cos(\quad + t)\, y(t)\, dt .$$ 8

(b) Solve : $y(x) = 1 + \quad (1 + e^{x+t})\, y(t)\, dt .$ 8

4. (c) Determine the eig values and eigen functions of the homogeneous tegral equation

$$y(x) = \lambda \quad 5xt^3 + 4x^2t)\, y(t)\, dt .$$ 8

(d) Find the solution of int· ral equation

$$y(x) = (1 + x)^2 + \quad xt + x^2t^2)\, y(t)\, dt .$$ 8

UNIT—III

5. (a) Prove that the resolvent kernel satisfies the integro-differential equation

$$\frac{\partial}{\partial\lambda}\left[R(x,t,\lambda)\right] = \int_a^b R(x, z, \lambda)\, R(z, t, \lambda)\, dz .$$

8

(b) Solve by the method of successive approximation,

$$y(x) = \frac{3}{2}e^x - \frac{1}{2}x\, e^x - \frac{1}{2} + \frac{1}{2}\int_0^1 t \cdot y(t)\, dt$$

8

6. (c) Define resolvent kernel and find the resolvent kernel for the Fredholm integral equation having kernel, $k(x, t) = e^{x+t}$; $a = 0$ and $b = 1$. 8

(d) Solve the integral equation,

$$y(x) = 1 + \lambda \int_0^1 (x + t)\, y(t)\, dt$$

by the method of successive approximation to third order. 8

UNIT—IV

7. (a) Find the resolvent kernel of the Volterra integral equation with the kernel $k(x, t) = e^{x^2 - t^2}$. 8

(b) Solve the Volterra integral equation of first kind,

$$f(x) = \int_0^x e^{x-t}\, y(t)\, dt , \quad f(0) = 0.$$ 8