AU–296

M.Sc. (Part—I) Semester—II (CBCS Scheme) Examination

203 : MATHEMATICS

(Integral Equation)

Time : Three Hours] [Maximum Marks : 80

Note :— Solve **ONE** question from each unit.

UNIT—I

1. (a) Show that the function $y(x) = xe^x$ is a solution of Volterra integral equation :

$$y(x) = \sin x + 2\int_0^x \cos(x - t)\, y(t)dt.$$ 8

(b) Convert the following differential equation into integral equation :

$$y'' + \lambda xy = f(x),\ y(0) = 1,\ \ y'(0) = 0.$$ 8

2. (c) Convert $y'' - \sin x\, y' + e^x y = x$ with initial condition $y(0) = 1$, $y'(0) = -1$ to the volterra integral equation of second kind. 8

(d) Show that the function $y(x) = 1 - x$ is a solution of the volterra integral equation

$$\int_0^x e^{x-t} y(t)dt = x.$$ 8

UNIT—II

3. (a) Solve the homogeneous Fredholm integral equation of second kind :

$$y(x) = \lambda \int_0^{2\pi} \sin(x + t)\, y(t)\, dt.$$ 8

(b) Determine the eigen values and eigen function of integral equation :

$$y(x) = \lambda \int_0^1 (2xt - 4x^2)\, y(t)\, dt.$$ 8

4. (c) Solve :

$$y(x) = e^x + \lambda \int_0^1 2e^x e^t y(t)dt .$$ 8

(d) Determine the eigen values and eigen function of homogeneous integral equation :

$$y(x) = \lambda \int_0^\pi (\cos^2 x \cos 2t + \cos 3x \cos t)\, y(t)\, dt .$$ 8

UNIT—III

5. (a) Solve by the method of successive approximation to third order by taking $y_0(x) = 1$.

$$y(x) = 1 + \lambda \int_0^1 (x + t)\, y(t)\, dt .$$ 8

(b) Find iterated kernel for the Fredholm integral equation having kernel :

$k(x, t) = e^x \cos t \quad a = 0, \quad b = \pi.$ 8

6. (c) Using iterative method, solve :

$$y(x) = f(x) + \lambda \int_0^1 e^{x-t} y(t)\, dt .$$ 8

(d) Find the resolvent kernel for the Fredholm integral equation having kernel :

$k(x, t) = (1 + x)(1 - t), \quad a = -1, \quad b = 1.$ 8

UNIT—IV

7. (a) Find the resolvent kernel of the Volterra integral equation with the kernel $k(x, t) = e^{x-t}$. 8

(b) Using the method of successive approximations, solve :

$$y(x) = 1 + x + \int_0^x (x - t)\, y(t)\, dt. \quad y_0(x) = 1 .$$ 8

8. (c) Using the method of successive approximation, solve :

$$y(x) = \frac{x^2}{2} + x - \int_0^x y(t)\,dt \quad \text{taking } y_0(x) = x.$$ 8

(d) Find the solution of integral equation with the help of resolvent kernel :

$$y(x) = e^{x^2} + \int_0^x e^{x^2} - t^2 \; y(t)\,dt\,.$$ 8

UNIT—V

9. (a) Examine whether a Green's function exists for the following b.v.p. :

$y'' = 0$, $y(0) = y(1)$ and $y'(0) = y'(1)$. 4

(b) Find the Green's function of the boundary value problem :

$y'' + \mu^2 y = 0$, $y(0) = y(1) = 0$. 12

10. Define Green's function and using Green's function, solve the boundary value problem :

$y'' - y = x$, $y(0) = y(1) = 0$. 4+12