AV–1945

M.Sc. (Part—I) Semester—II (CBCS Scheme) Examination

203 : MATHEMATICS

(Integral Equations)

Time : Three Hours] [Maximum Marks : 80

Note :— Solve **ONE** question from each unit.

UNIT—I

1. (a) For what value of λ, the function $y(x) = 1 + \lambda x$ is a solution of integral equation $x = \int_0^x e^{x-t}\, y(t)\, dt$? 8

   (b) Show that $\int_0^x y(t)\, dt^n = \int_0^x \frac{(x-t)^{n-1}}{(n-1)!}\, y(t)\, dt$. 8

2. (a) Define (i) Symmetric kernel, (ii) Separable kernel and show that $y(x) = xe^x$ is a solution of $y(x) = \sin x + 2\int_0^x \cos(x-t)\, y(t)\, dt$. 2+2+6

   (b) Show that $u(x) = -1 - \int_0^x (2x - t)\, u(t)\, dt$ is an integral equation corresponding to the DE $y'' + xy' + y = 0$ with initial condition $y(0) = 1$, $y'(0) = 1$, where $u(x)$ is an unknown function of x. 6

UNIT—II

3. (a) Solve $y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\, y(t)\, dt$. 8

   (b) Find the resolvent kernel of :

   $$y(x) = f(x) + \lambda \int_{-1}^{1} (xt + x^2 t^2)\, y(t)\, dt.$$ 8

4. (a) Find the eigen function of the homogeneous Fredholm integral equation of the second kind $y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)\, dt$. 8

   (b) Show that the homogeneous integral equation $y(x) = \lambda \int_0^1 \left(t\sqrt{x} - x\sqrt{t}\right)$ does not have real eigen values and eigen functions. 8

UNIT—III

5. (a) Find the resolvent kernel of the Volterra integral equation with kernel $k(x, t) = e^{x-t}$. 8

(b) Find Neumann series for the Volterra integral equation $y(x) = 1 + \int_0^x xt\, y(t)\, dt$. 8

6. (a) Using the method of successive approximation, solve the integral equation

$$y(x) = 1 + x - \int_0^x y(t)\, dt \text{, with } y(x) = 1$$ 8

(b) Find the Neumann series for the solution of integral equation $y(x) = 1 + x + \lambda \int_0^x (x - t)\, y(t)\, dt$.

8

UNIT—IV

7. (a) Solve $f(x) = \int_0^x e^{x-t}\, y(t)\, dt$, $f(0) = 0$, by converting it into Volterra integral equation of second kind. 8

(b) Using the method of successive approximation, solve the integral equation :

$$y(x) = 1 + \int_0^x (x - t)\, y(t)\, dt \text{, taking } y_0(x) = 1.$$ 8

8. (a) Change the Volterra integral equation of the first kind into the integral equation of second kind $\int_0^x \cos(x - t)\, u(t)\, dt = x$. 2

(b) Find the resolvent kernel of the Volterra integral equation with kernel $k(x, t) = 1$. 8

(c) Solve $y(x) = x + \int_0^x (t - x)\, y(t)\, dt$. 6

UNIT—V

9. Define the Green's function. Find the Green's function of boundary value problem $y'' = 0$, $y(0) = y(\ell) = 0$. 2+14

10. Construct Green's function for DE $xy'' + y' = 0$ for the following condition :

$y(x)$ is bounded as $x \to 0$ and $y(1) = \alpha y'(1)$, $\alpha = 0$. 16