(b) If P^r and Q^r are any two vectors in an elementary 2-space V_2, then show that :

$$k = \frac{R_{rsmn} P^r Q^s P^m Q^n}{(g_{rm} g_{sn} - g_{rn} g_{sm}) P^r Q^s P^m Q^n},$$

where k is a Riemannian curvature. 8

10. (c) Define isotropic point and show that at an isotropic point the Riemannian curvature satisfies

$$R_{rsmn} = k\,(g_{rm}\, g_{sn} - g_{rn}\, g_{sm}). \qquad 2+6$$

(d) Define Einstein space. Show that every Riemannian space V_2 is an Einstein space. 2+6

M.Sc. Part—I (Semester–II) (CBCS Scheme) Examination (Old)

MATHEMATICS

(Riemannian Geometry)

Paper—205 (Optional)

Time—Three Hours] [Maximum Marks—80

N.B. :— Attempt **ONE** question from each Unit.

UNIT—I

1. (a) Compute $\Gamma^1_{22}, \Gamma^2_{12}, \Gamma^3_{23}, \Gamma^4_{44}$ for metric

$$ds^2 = -e^A\, dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + e^B\, dt^2,$$

where A and B are functions of r alone. 8

(b) Show that the absolute derivative $\dfrac{\delta T^r}{\delta u}$ is contravariant vector. 8

2. (c) Show that the vectors $A^m = (1, 1, 1)$ and $B^m = (2, -1, -1)$ are orthogonal in space $ds^2 = dx^2 + dy^2 + dt^2$, but not in $ds^2 = dx^2 + dy^2 - dt^2$. 3+3

(d) Define :

(i) Christoffel symbols of first kind

(ii) Christoffel symbols of second kind

(iii) Riemannian Metric.

And prove transformation law for Christoffel symbols as

$$[mn, r]' = [ab, c]\frac{\partial x^a}{\partial x'^m}\frac{\partial x^b}{\partial x'^n}\frac{\partial x^c}{\partial x'^r} + g_{ab}\frac{\partial x^a}{\partial x'^r}\frac{\partial^2 x^b}{\partial x'^m \partial x'^n}.$$

2+2+2+4

UNIT—II

3. (a) Define geodesics and prove that geodesics are auto-parallel curves. 2+6

(b) Show that any field of parallel vectors is of constant magnitude. 8

4. (c) If two vectors, of constant magnitude, undergo parallel displacements along a given curve, then show that they are inclined at a constant angle. 8

(d) Describe Riemannian coordinates. Show that necessary and sufficient condition that the system x'^i be Riemannian coordinates is that the equations $\Gamma'^i_{mn} x'^m x'^n = 0$ hold throughout the space V_N. 2+6

UNIT—III

5. (a) For the metric in polar coordinates

$$ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2),$$

calculate the non-zero components of curvature tensor R_{prmn}. 8

(b) Show that :

(i) $R_{prmn} = R_{mnpr}$

(ii) $R_{prmn} + R_{pmnr} + R_{pnrm} = 0.$ 4+4

6. (c) Obtain expression for covariant curvature tensor. 8

(d) Prove that the number of independent components of curvature tensor R_{prmn} in N-dimensional space is $\frac{N^2(N^2-1)}{12}$. 8

UNIT—IV

7. (a) Show that divergence of Einstein tensor vanishes. 8

(b) State and prove Bianchi identity. 8

8. (c) Define Ricci tensor and Einstein tensor. Also show that each of which is symmetric tensor. 2+2+2+2

(d) Find the equation of Geodesic deviation. 8

UNIT—V

9. (a) Define Riemann curvature and show that it is independent of the pair of orthogonal unit vectors in elementary 2-space V_2. 2+6