AQ–886

M.Sc. Part—I (Semester—II) (CBCS Scheme) Examination (New)

MATHEMATICS

(Riemannian Geometry)

Paper—205 (Optional)

Time—Three Hours] [Maximum Marks—80

Note :— Solve **FIVE** questions, selecting **ONE** from each Unit.

UNIT—I

1. (a) Define Riemann Space. Show that an element of volume $g^{1/2}dx^1dx^2 \ldots dx^N$ is invariant in coordinate transformation. 2+6

(b) Calculate the components of the fundamental metric tensor g_{mn} and its conjugate g^{mn} for the given metric :

$$ds^2 = 2(dx^1)^2 + 4(dx^2)^2 - (dx^3)^2 + 6dx^1dx^2 - 8dx^2dx^3.$$

8

2. (c) Define absolute derivative and show that $\frac{\delta T_r}{\delta u}$ is a covariant vector. 2+8

(d) Find the value of k, if the vector $A^m = \left(1, \frac{k}{r}, 0, 2\right)$ in the space with metric $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta - dt^2$ is unit. 6

UNIT—II

3. (a) Define parallel vector field in a Riemannian Space V_N. Show that any field of parallel vectors is of constant magnitude. 2+6

   (b) Derive differential equation of geodesics. 8

4. (c) If two vectors of constant magnitudes, undergo parallel displacement along a given curve, then show that they are inclined at a constant angle. 8

   (d) Define geodesic null line. Show that $x = au + a'$, $y = bu + b'$, $z = cu + c'$ may be represented as the geodesic null lines equations in three space with coordinates x, y, z and metric $ds^2 = dx^2 + dy^2 - dz^2$, where u is a parameter and a, a′, b, b′, c, c′ are constants which are arbitrary except for the relation $a^2 + b^2 - c^2 = 0$. 2+6

UNIT—III

5. (a) List the independent components of Riemannian tensor R_{rpmn} for the Riemannian space V_N, N = 2, 3, 4. 8

   (b) Show that the number of independent components of R_{prmn} in N-dimensional space is $\frac{N^2(N^2-1)}{12}$. 8

6. (c) State and prove cyclic properties of curvature tensor. 2+3+3

   (d) Obtain an expression for Riemann-Christoffel curvature tensor of the second kind. 8

UNIT—IV

7. (a) Define Ricci tensor and Einstein tensor and show that they are symmetric tensor. 2+2+2+2

   (b) State and prove Bianchi identity. 2+6

8. (c) Obtain an expression for geodesics deviation. 8

   (d) Define Einstein tensor and show that the divergence of Einstein tensor vanishes. 2+6

UNIT—V

9. (a) Define :

   (i) Riemannian curvature

   (ii) Isotropic point

   (iii) Einstein space

   (iv) Flat space. 2+2+2+2

   (b) Show that every Riemannian space V_2 is an Einstein space. 8

10. (c) If $R_i^a = g^{am} R_{im}$, then show that $R^a_{i;a} = \frac{1}{2}\frac{\partial R}{\partial x^i}$ and deduce that, when N > 2, the scalar curvature of an Einstein space is constant. 4+4

   (d) Prove that the Riemannian curvature is independent of the pair of orthogonal unit vectors in V_2. 8