AQ–878

M.A./M.Sc. (Semester—II) (CBCS Scheme) Examination (Old)

MATHEMATICS–204

(Topology—II)

Time—Three Hours]   [Maximum Marks—80

Note :— Attempt **ONE** question from each Unit.

UNIT—I

1. (a) Define metric topology. Let X be any non-empty set and the mapping $d^* : X \times X \to R$ is defined by

$$d^*(x, y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$$

Then show that d^* is a metric for X. 8

(b) Show that each ball is an open set for the relative topology. 8

2. (c) Show that every Lindelöf metric space is a second axiom. 8

(d) Show that the space (C, d_c) of all real-valued continuous functions is separable. 8

UNIT—II

3. (a) Prove that the space C of continuous functions is complete. 8

(b) Define the following :

(i) Complete enclosure

(ii) Embedded metric space

(iii) Absolutely closed metric space

(iv) Nested sequence. 8

4. (c) Prove that metric space is complete if and only if every infinite totally bounded subset has a limit point. 8

(d) Show that a metric space is complete if and only if it is absolutely closed. 8

UNIT—III

5. (a) If (X, d_x) and (Y, d_y) are metric spaces then show that the function

$$d(<x_1, y_1>, <x_2, y_2>) = \sqrt{d_x^2(x_1, x_2) + d_y^2(y_1, y_2)}$$

is a metric for $X \times Y$ which induces the product topology. 8

(b) If H is a Hilbert space then show that $H \times H$ is isometric to H. 8

6. (c) Define metric products and perfect set. Show that a product metric space D is perfect. 8

(d) Prove that $\Pi_\lambda X_\lambda$ is Hausdorff space if and only if each space X_λ is Hausdorff. 8

UNIT—IV

7. (a) Define :

(i) Point-open topology

(ii) Evaluation mapping

(iii) Induced topology

(iv) Compact-open topology. 8

(b) If Y is regular then show that the set of all continuous mappings of X into Y, I(X, Y) with the compact open topology is regular. 8

8. (c) Prove that a subset G of Y is open in the quotient topology relative to $F : X \to Y$ if and only if $f^{-1}(G)$ is an open subset of X. 8

(d) Show that the quotient topology with Y is a T_1-space if and only if $f^{-1}(y)$ is closed in X for every $y \in Y$. 8

UNIT—V

9. (a) State and prove Stone's theorem. 8

(b) Show that every Paracompact regular space is normal. 8

10. (c) Define generalized Hilbert space and prove that in T_3-space with a σ-locally finite base, every open set is an F_σ set. 8

(d) State and prove Nagata-Smirnov metrization theorem. 8