(b) Show that paracompactness is a topological property. 6

10. (c) State and prove Nagata Metrization theorem. 12

(d) Show that in a T_3-space with a 6-locally finite base, every open set is an F_6 set. 4

AQ–885

M.Sc. (Semester – II) (CBCS Scheme) Examination

MATHEMATICS (New)

Topology–II

Time—Three Hours] [Maximum Marks—80

N.B. :— Attempt **one** question from each unit.

UNIT–I

1. (a) Define induced topology. Show that the family of all balls of points in a set X with metric d forms a base for a topology for X. 8

(b) Define Lindelöf space and show that Lindelöf metric space is second axiom. 8

2. (c) Show that, every separable metric space is homeomorphic to some subset of Frechet space. 8

(d) Define completely normal space and show that every metric space is completely normal. 8

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UNIT–II

3. (a) Prove that metric space is complete if and only if it is absolutely closed. 8

(b) Show that, every closed subset of a complete metric space is complete. 8

4. (c) Show that the metric space is complete iff the intersection of every nested sequence of nonempty closed balls with radii tending to zero is nonempty. 8

(d) Define Cauchy sequence and show that the space of Frechet space is complete. 8

UNIT–III

5. (a) Prove that $\Pi_\lambda X_\lambda$ is Hausdorff if and only if each X_λ is Hausdorff. 8

(b) Define filter and ultrafilter. Show that every filter is contained in an ultrafilter. 8

6. (c) Show that if (X,dx) and (Y,dy) are metric spaces, then the function d defined by setting

$d(\langle x_1 y_1\rangle,\langle x_2 y_2\rangle)=\sqrt{dX^2(x_1,x_2)+dY^2(y_1 y_2)}$ is a metric for X × Y which induces the product topology. 8

(d) Prove that X × Y is dense-in-itself iff at least one of the spaces X and Y is dense-in-itself. 8

UNIT–IV

7. (a) If $\langle fn\rangle$ is a sequence of points in F (X,Y) with the topology of pointwise convergence, then show that $\lim f_n = f$ iff $\lim f_n(x) = f(x)$ for every $x \in X$. 8

(b) If Y is regular, then $\tau(X,Y)$, with the compact-open topology, is regular. Prove this. 8

8. (c) Prove that if X is compact, connected, locally connected, separable, then so is Y with the quotient topology. 8

(d) Show that Y, with the quotient topology is T_1-space iff F^{-1} (Y) is closed in X for every $y \in Y$. 8

UNIT–V

9. (a) Prove that in a regular space X, the following conditions are equivalent to paracompactness :

(i) For every open covering of X, there is a 6-locally finite open cover which refines it.

(ii) For every open covering of X, there is a 6-discrete open covering which refines it.

(iii) For every open covering of X, there is an open cover which star refines it. 10