AU–297

M.Sc. Semester–II (CBCS Scheme) Examination

MATHEMATICS

(Topology–II)

Paper–204

Time : Three Hours] [Maximum Marks : 80

N.B. :— Attempt **one** question from each unit.

UNIT–I

1. (a) Prove that the family of all balls of points in a set X with metric d forms a base for a topology for X. 8

   (b) Prove that every separable metric space is second axiom. 8

2. (c) Define Hilbert space (H, d_H). Prove that Hilbert space is not locally compact. 2+6

   (d) Define Hilbert cube and show that co-ordinatewise and pointwise convergence are equivalent in Hilbert cube. 2+6

UNIT–II

3. (a) Prove that Hilbert space is complete. 8

   (b) Prove that every metric space is isometric to a dense subset of a complete metric space. 8

4. (c) Prove that a metric space is complete iff it is absolutely closed. 8

   (d) Prove that in a complete metric space the intersection of a countable number of dense, open sets is itself dense. 8

UNIT–III

5. (a) Prove that $H \times H$ is an isometric to H. 8

   (b) Prove that $X \times Y$ is compact iff X and Y are compact. 8

6. (c) For each $x, y \in K$, let $x = \sum_{i=1}^{\infty} x_i / 3^i$ and $y = \sum_{i=1}^{\infty} y_i / 3^i$, show that $x = y \Leftrightarrow x_i = y_i$ for all i. 8

   (d) If D is subset of Frechet space and K is canter discontinuum then prove that D is homeomorphic to K. 8

UNIT–IV

7. (a) Define :

   (i) Point-open topology. 2

   (ii) Compact-open topology. 2

   (iii) Evaluations. 2

   (iv) Pseudometric space. 2

   (b) Prove that Y with the quotient topology is T_1–space iff $f^{-1}(y)$ is closed in X for every $y \in Y$. 8

8. (c) Prove that if a subset G of Y is open in quotient topology iff $f^{-1}(G)$ is an open subset of X. 8

   (d) Show that if (X, d) is a pseudometric space then the relation τ defined by setting $\langle x, y \rangle \in \tau$ iff $d(x, y) = 0$, is an equivalence relation and the quotient space X/τ is metrizable. 8

UNIT–V

9. (a) State and prove Urysohn's Lemma. 8

   (b) Define Paracompact. Prove that every paracompact regular space is normal. 2+6

10. (c) Prove that for every open covering of regular topological space X, there is a locally finite open cover which refines it iff there is locally finite cover which refines it. 8

    (d) (i) Define Generalized Hilbert space (H^{τ}, dH^{τ}).

    (ii) Write statement of Bing metrization theorem. 4

    (e) Prove that a T_3–space with σ–locally finite base is normal. 4