AV–1946

M.Sc. (Semester—II) (CBCS Scheme) Examination

204 : MATHEMATICS

(Topology—II)

Time : Three Hours] [Maximum Marks : 80

N.B. :— Attempt **ONE** question from each unit.

UNIT—I

1. (a) Define Frechet Space (F, d_F). Show that if, in F, let $X_k = \langle x_1^k, x_2^k, \ldots \rangle$ and $X = \langle x_1, x_2, \ldots \rangle$, then $\lim_k X_k = X$ iff $\lim_k x_i^k = x_i \ \forall\, i \in N$. 2+6

   (b) Define metric space. Show that every metric space is a Hausdorff space. 2+6

2. (c) Show that the space of continuous functions on I = [0, 1] is separable. 8

   (d) Define ε-net for a subset E of a metric space. Show that every countably compact metric space is separable. 2+6

UNIT—II

3. (a) Define Cauchy sequence and show that Hilbert space is complete. 2+6

   (b) Show that all completions of a metric space are isometric. 8

4. (c) Define :

   (i) Absolutely closed metric space

   (ii) Contraction map on a metric space

   (iii) First category set

   (iv) Completion of a metric space. 2+2+2+2

   (d) Show that, a metric space is complete iff every infinite totally bounded subset has a limit point. 8

UNIT—III

5. (a) Show that $\prod_\lambda X_\lambda$ is Hausdorff if each X_λ is Hausdorff. 8

   (b) Show that X × Y is compact iff X and Y are compact. 8

6. (c) Define filter and ultrafilter. Prove that every filter is contained in an ultrafilter. 2+2+4

   (d) Prove that $\prod_\lambda X_\lambda$ is connected iff each space X_λ is connected. 8

UNIT—IV

7. (a) If $\langle f_n \rangle$ is a sequence of points in $\mathfrak{F}(X, Y)$ with the topology of pointwise convergence, then show that $\lim f_n = f$ iff $\lim f_n(x) = f(x)$ for every x in X. 8

   (b) If Y is a T_0 space, then show that $\mathfrak{F}(X, Y)$ is a T_0 space with the compact open topology. 8

8. (c) Show that, a subset G of Y is open in the quotient topology (relative to $f : X \to Y$) iff $f^{-1}(G)$ is an open subset of X. 8

(d) Show that Y, with the quotient topology, is a T_1 space iff $f^{-1}(y)$ is closed in X for every y in Y. 8

UNIT—V

9. (a) State :

(i) Nagata-Smirnov Metrization Theorem

(ii) Urysohn's Lemma

(iii) Urysohn's Metrization Theorem

(iv) Bing Metrization Theorem. 8

(b) Define σ-locally finite family. Show that in a T_3-space with a σ-locally finite base, every open set is an F_σ-set. 2+6

10. (c) Show that paracompactness is a topological property. 8

(d) Show that every paracompact regular space is normal. 8