AQ–909

M.A./M.Sc. (Semester—II) Examination

(CBCS Scheme)

STATISTICS

Paper—V

(Advanced Probability Theory)

Time—Three Hours] [Maximum Marks—80

Note :— Answer either A or B in each question.

1. (A) (a) State and prove Borel Cantelli Lemma.

(b) Define random variable. State and prove Markov Inequality. 12+4

OR

(B) (i) State and prove Cauchy Schwartz Inequality.

(ii) State and prove multiplication theorem of expectation of random variable. 8+8

2. (A) (a) Show that operation of convergence in probability is closed under arithmetic operation.

(b) Show that convergence in r^{th} mean implies convergence in probability. 8+8

OR

(B) (i) Define :

(a) Convergence in law

(b) Convergence almost sure

(c) Convergence in probability.

(ii) Define distribution function. Prove any two properties of distribution function. 9+7

3. (A) (a) State and prove Inversion theorem.

(b) Find p.d.f. of random variable whose characteristics function is given by :

$\phi(t) = e^{-|t|}$. 8+8

OR

(B) (i) State and prove any three properties of characteristic function.

(ii) Obtain characteristic function of normal distribution. 8+8

4. (A) (a) State and prove Bernoulli's law of large numbers.

(b) Let $\{X_n\}$ be a sequence of independent random variables.

$$\text{If } \sum_{n=1}^{\infty} \frac{V(X_n)}{B_n^2} < \infty, B_n \uparrow \infty$$

then prove that :

$$\frac{S_n - E(S_n)}{B_n} \xrightarrow{a,s,} 0.$$ 8+8

OR

(B) (i) State and prove Khinchin's law of large numbers.

(ii) State and prove necessary and sufficient conditions for a sequence to obey WLLN. 8+8

5. (A) State and prove De-Moivere's Laplace central limit theorem. 16

OR

(B) State and prove Lyapounon's central limit theorem. 16