AV–1974

M.A./M.Sc. (Semester—II) (CBCS Scheme) Examination

STATISTICS

(Advanced Probability Theory)

Paper—V

Time : Three Hours] [Maximum Marks : 80

Note :— Answer either (A) or (B) in each question.

1. (A) (a) Define :

(i) Random variable

(ii) Probability space

(iii) Expectation of random variable.

(b) State and prove Borel-Cantelli Lemma. 6+10

OR

(B) (p) Define mutual independence and pairwise independence of three events. Show that pairwise independence does not imply mutual independence.

(q) State and prove Chebyshev's inequality. 8+8

2. (A) (a) Define convergence in probability. Show that convergence in probability implies convergence in distribution.

(b) Let $\{Y_n\}$ be an arbitrary sequence of r.v. then Y_n converges to zero in probability iff

$$E\left[\frac{|Y_n|^r}{1+|Y_n|^r}\right] \to 0 \text{ for some } r > 0 \text{ as } n \to \infty.$$ 8+8

OR

(B) (p) Define convergence almost sure and convergence in probability for a sequence of random variables.

(q) Show that $X_n \xrightarrow{a.s} X$ iff :

$$\lim_{n \to \infty} P\left[\sup_{m \geq n} |X_m - X| > \epsilon\right] = 0 \text{ for some } \epsilon > 0.$$

(r) Define convergence in r^{th} mean. Show by means of an example convergence in probability does not imply convergence in r^{th} mean. 5+5+6

3. (A) (a) Define characteristic function of a random variable. State and prove any three properties of characteristic function.

(b) State and prove Inversion theorem. Obtain probability density function of a random variable whose characteristic function is given by $\phi_X(t) = e^{i\mu t - \frac{1}{2}\sigma^2 t^2}$. 6+10

OR

(B) (p) Comment on the following statements :

(i) $\phi_{X+Y}(t) = \phi_X(t) \cdot \phi_Y(t)$ implies X and Y are independent.

(ii) Characteristic function is affected by change of origin and scale.

(q) Obtain the characteristic function of a Cauchy distribution with probability density function as $f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$.

(r) Find characteristic function of Poisson distribution. 8+5+3

4. (A) (a) Distinguish between weak and strong law of large numbers for a sequence of random variables.

(b) State and prove necessary and sufficient condition for a sequence to obey WLLN. 8+8

OR

(B) (p) State and prove Khinchin's law of large numbers.

(q) State Kolmogorov's strong law of large numbers. 8+8

5. (A) (a) State and prove Liapounoff's Central Limit theorem. 16

OR

(B) (p) Explain multivariate form of Central Limit theorem.

(q) Explain the role of Central Limit theorem in different fields. 8+8