M.Sc. (Semester—II) (CBCS Scheme) Examination
STATISTICS
(Testing and Hypothesis)
Paper—VI

Time : Three Hours] [Maximum Marks : 80

Note :— Answer either (A) or (B) from each question.

1. (A) (i) Define the terms :
   (a) Null hypothesis
   (b) Alternate hypothesis
   (c) Critical region.

   (ii) Write the statement of NP lemma and prove its sufficiency part. 6+10

OR

(B) (i) Let $x_1, x_2, ...x_{25}$ are iid r.v. with $N(\theta, 100)$ for testing hypothesis $H_0 : \theta = 75$ against $H_1 : \theta = 80$. The following test procedure is used :

$$\phi(x_1, x_2, ... x_{25}) = \begin{cases} 1, & \text{if } \bar{x} \geq 75 \\ 0, & \text{if } \bar{x} < 75 \end{cases}$$

Find size of test and power of test.

(ii) Describe MP test and UMP test. 8+8

2. (A) (i) Define monotone likelihood ratio (MLR) property with example.

   (ii) State and prove Karlin Rubin theorem. 6+10

OR

(B) (i) Prove that UMP test does not exists for testing the hypothesis $H_0 : \theta_1 \leq \theta \leq \theta_2$ against $H_1 : \theta < \theta_1$ or $\theta > \theta_2$, even in case of one parameter exponential family possess MLR property.

(ii) For the r.s. of size n from $N(\mu, \sigma^2)$, construct UMP α level test for testing the hypothesis $H_0 : \mu \leq \mu_0$ against $H_1 : \mu > \mu_0$ where σ^2 is known. 8+8

3. (A) (i) Construct LR test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, when r.s. has been drawn from normal population $N(\mu, \sigma^2)$, where μ and σ^2 are unknown. Find the critical region.

   (ii) Describe Bartlett's test. 8+8

OR

(B) (i) Show that for given α, $0 \leq \alpha \leq 1$, if a non randomised NP test and LR test for testing simple hypothesis against simple alternate hypothesis exist, then they are equivalent.

(ii) Describe Pearson's χ^2 test for goodness of fit. 8+8

4. (A) (i) Establish the relationship between the parameters of SPRT.

(ii) Define SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$ where θ is the parameter of Poisson distribution. Also find OC curve. 8+8

OR

(B) (i) Prove that SPRT terminates with probability 1.

(ii) Describe sequential probability ratio test. 8+8

5. (A) (i) Define :

(a) Unbiased test

(b) Completeness

(c) Bounded completeness.

(ii) State and prove necessary and sufficient condition for every similar test to have Neyman structure. 6+10

OR

(B) (i) Show that every UMP test is unbiased test.

(ii) If $X \sim b(n, p)$, construct UMPU test of size α for testing $H_0 : p = p_0$ against $H_1 : p \neq p_0$. 8+8