M.A./M.Sc. Semester—II (CBCS Scheme) Examination

STATISTICS

(Testing of Hypothesis)

Paper—VI

Time : Three Hours] [Maximum Marks : 80

Note :— Answer either (A) or (B) in each question.

1. (A) (i) Define the following terms :

   (a) Critical region

   (b) Two types of error

   (c) Level of significance

   (d) Power of test.

   (ii) Define MP test and UMP test. 8+8

OR

   (B) (i) Describe p-value concept in testing of hypothesis. Compare it with critical value concept.

   (ii) State NP lemma and prove its necessary part. 6+10

2. (A) (i) Construct UMP α level test for testing the hypothesis $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$ for B(n, θ), where n is known and θ is unknown and let x be the random variable representing no. of successes.

   (ii) Define Monotone Likelihood Ratio (MLR) property and show that $X \sim U(0, \theta)$ with $\theta > 0$ has MLR property in T(x). 8+8

OR

   (B) (i) State and prove Karlin-Rubin theorem.

   (ii) Obtain UMP α level test of testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$, where the r.s. of size n are from Poisson distribution with unknown parameter θ. 10+6

3. (A) (i) Construct LR test of size α for $X \sim B(n, p)$ to test the hypothesis $H_0 : P \leq P_0$ against $H_1 : P > P_0$.

(ii) Describe Pearson's χ^2 test for goodness of fit. 8–8

OR

(B) (i) Given a r.s. of size n from the population with pdf :

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & ; 0 < x < \infty \\ 0 & ; \text{otherwise} \end{cases}$$

Obtain LR test of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(ii) Describe Wald's test. 8–8

4. (A) (i) Prove that SPRT terminates with probability one.

(ii) Define SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (\theta_1 > \theta_0)$, where θ is the parameter of Poisson distribution. Find the expression for OC function. 8–8

OR

(B) (i) Obtain SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (\theta_1 > \theta_0)$. The samples are drawn from normal distribution where σ is known. Also obtain ASN function.

(ii) Describe OC function for SPRT. 8+8

5. (A) (i) Define :

(a) Unbiased test

(b) Completeness

(c) Bounded completeness.

(ii) State and prove necessary and sufficient condition for every similar test to have Neyman structure 6–10

OR

(B) (i) For $X \sim N(\theta, \sigma^2)$, construct UMPU test for testing $H_0 : \theta_1 < \theta < \theta_2$ against $H_1 : \theta < \theta_1$ or $\theta > \theta_2$.

(ii) Show that every UMP test is unbiased test. 8+8