AU–374

M.Sc. (Part—II) Semester—III Examination

MATHEMATICS (Old) upto S/18

(302 : Advanced Mechanics)

Time : Three Hours] [Maximum Marks : 80

Note :— Attempt any **ONE** question from each Unit.

UNIT—I

1. (a) Define :
   (i) Extremum points
   (ii) Stationary point
   (iii) Functional on class of functions
   (iv) Variation Sf. 8
   (b) Show that the distance between any two points in a plane is a straight line. 8
2. (c) Derive the Lagrange's equations of motion for a conservative system. 8
   (d) Obtain the Lagrangian equation of motion for the double pendulum of lengths ℓ_1 and ℓ_2 with corresponding masses m_1 and m_2. 8

UNIT—II

3. (a) Define :
   (i) Generalized Coordinates
   (ii) Generalized Displacement
   (iii) Holonomic Constraints
   (iv) Rheonomic Constraints. 8
   (b) State and prove D'Alembert's principle. 8
4. (c) Derive he Lagrange's equations of motion for Non-Holonomic Conservative System. 8
   (d) Derive the Lagrange's equations of motion in the form :

$$\left(\frac{\partial \dot{T}}{\partial \dot{q}_i}\right) - 2\left(\frac{\partial T}{\partial q_i}\right) = Q_i .$$ 8

UNIT—III

5. (a) Define cyclic coordinates and prove that the generalized momentum conjugate to a cyclic coordinate is conserved. 8

(b) Prove that $\frac{dH}{dt} = -\frac{\partial L}{\partial t}$. 8

6. (c) Derive the Hamilton's equations of motion :

$$\dot{q}_i - \frac{\partial H}{\partial p_i} = 0 \text{ and}$$

$$\dot{p}_i - \frac{\partial H}{\partial q_i} = 0.$$ 8

(d) If the component of the applied torque vanishes then show that the corresponding component of the angular momentum is conserved. 8

UNIT—IV

7. (a) Explain the following terms :

(i) Canonical transformation

(ii) Generating function. 8

(b) Show that Lagrange's brackets are invariant under contact transformation. 8

8. (c) If F and G are two arbitrary functions then show that :

$$[F, G]_{q, p} = [F, G]_{Q, P}.$$ 8

(d) Show that if the Hamiltonian and a quantity F are constants of the motion then $\frac{\partial F}{\partial t}$ must be a constant. 8

UNIT—V

9. (a) Define Hamilton's principle function and write a note on the Hamilton-Jacobi equation. 8

(b) Using action-angle variables determine the frequency of the linear harmonic oscillator. 8

10. (c) Find the motion of a particle in a plane under the action of a central force. 16