M.Sc. (Part—II) Semester—III (CBCS) Examination

302 : MATHEMATICS

(Advanced Mechanics)

Time : Three Hours] [Maximum Marks : 80

Note :— Attempt **ONE** question from each unit.

UNIT—I

1. (a) Derive the Hamilton's canonical equations of motion from Hamiltonian function. 4

(b) Obtain Lagrangian L from Hamiltonian H and show that it satisfies Lagrange's equation of motion. Prove also that the Hamiltonian H thus defined also satisfies the Hamilton's canonical equation of motion. 8

(c) Show that $P = \frac{1}{2}(p^2 + q^2), Q = \tan^{-1}(q/p)$ is canonical. 4

2. (d) Show that the transformation defined by $e^Q = \frac{1}{q} \sin p$ and $P = q \cot p$ is canonical transformation.

Further, if Hamiltonian $H = \frac{p^2}{2m} + \frac{kq^2}{m}$, find the generating function $F_1(q, Q, t)$. 8

(e) If the Lagrangian L of the conservative system does not contain time t explicity, then prove that the Hamiltonian H is an integral of motion and is the total energy of the system. 8

UNIT—II

3. (a) Prove that Poisson brackets are invariant under canonical transforation

i.e. $[u, v]q, p = [u, v]_{Q, P}$,

where u and v are arbitrary functions. 8

(b) For the Hamiltonian $H = \frac{1}{2}(p^2 + q^2)$, find $[\dot{p}, H]$ and $[\dot{q}, H]$, solve the equation of motion and show that energy is conserved. 8

4. (c) State and prove Liouville's theorem. 8

(d) Define Poisson bracket and show that :

(i) $[u + v, w] = [u, w] + [v, w]$

(ii) $[u, vw] = v[u, w] + w[u, v]$. 8

UNIT—III

5. (a) Prove that the Hamilton's principle function is the generator of a canonical transformation to constant coordinates and momenta. 8

(b) Define action variable and angle variable. By using action angle variables determine the frequency of the linear harmonic oscillator. 8

6. (c) Use the Hamilton-Jacobi method to find the motion of particle falling vertically in a uniform gravitational field. 8

(d) In Kepler's problem show that :

$$J_\theta = \oint \sqrt{\alpha_\theta^2 - \frac{\alpha_\phi^2}{\sin^2\theta}}\, d\phi .$$ 8

UNIT—IV

7. (a) Derive the expression for effective potential i.e. :

$$v(\theta) = Mg\ell \cos\theta + \frac{I_1}{2}\left(\frac{b - a\cos\theta}{\sin\theta}\right)^2$$

in a heavy symmetrical top with one point fixed. 8

(b) Find the kinetic energy of a rigid body rotating about a fixed point of the body when the moments of inertia and product of inertia of the body relative to the set of axes through fixed point are known. 8

8. (c) Find a real matrix of orthogonal transformation in the 3-dimensional space corresponding to the unitary matrix :

$$Q = \begin{pmatrix} \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$ 8

(d) Show directly by vector manipulation that the definition of the moment of inertia as :

$$I = m_i(\bar{r}_i \times \bar{n}) \circ (\bar{r}_i \times \bar{n})$$

reduces to equation

$$I = \bar{n} \circ \dot{I} \circ \bar{n} = m_i(\bar{r}_i^2 - (\bar{r}_i \circ \bar{n})^2).$$ 8

UNIT—V

9. (a) By method of time dependent perturbation theory carry the solution for the linear harmonic oscillator out through third order terms, assuming the initial condition $\beta_0 = 0$. Find expressions for both x and p as a function of time. 10

(b) Write short notes on time dependent perturbation technique. 6

10. (c) By using the formulation $w = \frac{\partial Y}{\partial J} = w_0 + \varepsilon\frac{\partial Y_1}{\partial J}$ find the first order correlation in the dependence of θ on time. 8

(d) Find the precession rate by first order perturbation theory by considering general form for the perturbing potential of the form :

$$V = -\frac{k}{r} - \frac{h}{r^n}$$

where $n \geq 2$ is an integer. 8