AU–379

M.Sc. (Part-II) Semester-III (CBCS) Examination

MATHEMATICS

(Difference Equations—I)

Paper—306

Time : Three Hours] [Maximum Marks : 80

Note :— Solve any **ONE** question from each unit.

UNIT—I

1. (a) Prove that :

(i) $\Delta_t t^{\underline{r}} = r t^{\underline{r-1}}$

(ii) $\Delta_t \binom{t}{r} = \binom{t}{r-1}$, $r \neq 0$. 8

(b) Find the solution of the difference equation :

$$y(t+1) - y(t) = t^3 + 3^t.$$ 4

(c) Use the summation by part formula to evaluate :

$$\Sigma \binom{t}{2}\binom{t}{7}.$$ 4

2. (d) Compute $\Delta(3^t \cos t)$ by methods :

(i) Using formula $\Delta(y(t)\ z(t))$

(ii) Directly from the definition of Δ. 8

(e) Find the solution of the difference equation :

$$y(t+2) - 2y(t+1) + y(t) = \binom{t}{5}.$$ 4

(f) Use the summation by parts formula to evaluate :

$\Sigma t^2 3^t$. 4

UNIT—II

3. (a) Find the solutions of the difference equation :

$$u(t+3) - 6u(t+2) + 11u(t+1) - 6u(t) = 0$$

show that these solutions are linearly independent. 8

(b) Solve by annihilator method the difference equation :

$$y(t+2) - 7y(t+1) + 6y(t) = t.$$ 8

4. (c) Suppose $y(1) = 2$ and find the solution of $y(t+1) - 3y(t) = e^t$, $t = 1, 2, 3, \ldots\ldots$ 8

(d) Solve the Riccati equation :

$$y(t+1)\,y(t) + 2y(t+1) + 7y(t) + 20 = 0.$$ 8

UNIT—III

5. (a) Solve the first order initial value problem :

$$y_{k+1} + 3y_k = 4\delta_k(2),\ y_0 = 2$$

by using z-transform. 8

(b) If $Y(z) = Z(y_k)$ for $|z| > r$, then prove that :

$$Z((k+n-1)^{\underline{n}}\, y_k) = (-1)^n z^n \frac{d^n Y}{dz^n}(z),$$

for $|z| > r$. 8

6. (c) Solve the second order initial value problem :

$$y_{k+2} + y_k = 10\,3^k,\ y_0 = 0,\ y_1 = 0.$$ 8

(d) If $Z(y_k)$ exists for $|z| > r$, then prove that :

$$Z\left(\sum_{m=0}^{k} y_m\right) = \frac{z}{z-1} Z(y_k)$$

for $|z| > \max(1, r)$. 8

UNIT—IV

7. (a) Solve the difference equation :

$$u(t+1)=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}u(t)+\begin{bmatrix} t^2 \\ 0 \\ t \end{bmatrix}u(0)=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$ 8

(b) Find the spectrum and the spectral radius of :

$$\begin{bmatrix} 2 & -6 & -6 \\ -1 & 1 & 2 \\ 3 & -6 & -7 \end{bmatrix}.$$ 8

8. (c) Solve the initial value problem :

$$u(t+1)=\begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}u(t),\ u(0)=\begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$ 8

(d) Compute A^t, $t \geq 0$, if $A=\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ where the complex eigen values of A are $\lambda = 1 \pm i$. 8

UNIT—V

9. (a) Prove that the solution of difference equation $y_{n+1} - ny_n = 1$, $n = 1, 2, 3, \ldots\ldots$ is :

$$y_n=(n-1)!\left[y_1+\sum_{k=1}^{n-1}\frac{1}{k!}\right], n \geq 2.$$ 8

(b) Obtain the asymptotic behaviour of $\sum_{k=1}^{n} k^k$. 8

10. (c) Given that $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}=\frac{\pi^2}{12}$, show that $\sum_{k=1}^{n-1}\frac{(-1)^{k+1}}{k^2}=\frac{\pi^2}{12}+0\left(\frac{1}{n^2}\right)$, $n \to \infty$. 8

(d) Find an asymptotic approximation for $\sum_{k=1}^{n-1} a^k \log k, \text{as } n \to \infty$, if $a > 1$. 8