AU–376

M.Sc. (Part—II) Semester—III (CBCS) Examination

303 : MATHEMATICS

(Operations Research)

Time : Three Hours] [Maximum Marks : 80

Note :— Solve any **ONE** question from each Unit.

UNIT—I

1. (a) Use Simplex Method to solve the LPP :

Maximize $Z = 5x_1 + 3x_2$

subject to : $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0.$ 8

(b) Use Big-M Method to:

Maximize : $Z = 3x_1 - x_2$

subject to :

$2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0.$ 8

2. (c) Prove that dual of the dual of a given primal, is the primal itself. 8

(d) Use Dual Simplex method to :

Maximize : $Z = -2x_1 - x_2$

subject to :

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0.$ 8

UNIT—II

3. (a) Use branch and bound method to :

Minimize $Z = 4x_1 + 3x_2$

subject to :

$$5x_1 + 3x_2 \geq 30$$
$$x_1 \leq 4$$
$$x_2 \leq 6$$
$x_1, x_2 \geq 0$ and are integers. 8

(b) Using cutting plane algorithm :

Maximize : $Z = x_1 - x_2$

subject to :

$$x_1 + 2x_2 \leq 4$$
$$6x_1 + 2x_2 \leq 9$$
$x_1, x_2 \geq 0$ are non-negative integers. 8

4. (c) Using bounded variable technique, solve the following LPP :

Maximize $Z = 3x_1 + 5x_2 + 2x_3$

subject to :

$$x_1 + 2x_2 + 2x_3 \leq 14$$
$$2x_1 + 4x_2 + 3x_3 \leq 23$$
$$0 \leq x_1 \leq 4,\ 2 \leq x_2 \leq 5$$
$0 \leq x_3 \leq 3.$ 8

(d) Use simplex method to solve the following Goal programming problem :

Minimize $Z = p_1d_1 + p_2d_2 + 2p_2d_2 + p_3d_1^+$

subject to :

$$10x_1 + 10x_2 + d_1^- - d_1^+ = 400$$
$$x_1 + d_2^- = 40$$
$$x_2 + d_3^- = 30$$
$x_1, x_2, d_1^-, d_1^+, d_2^-, d_3 \geq 0.$ 8

UNIT—III

5. (a) Perform complete parametric analysis of maximize :

$$Z = (\lambda - 1)\, x_1 + x_2$$

subject to :

$$x_1 + 2x_2 \le 10$$
$$2x_1 + x_2 \le 11$$
$$x_1 - 2x_2 \le 3$$
$$x_1, x_2 \ge 0.$$ 8

(b) Solve the following assignment problem :

$$\begin{bmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{bmatrix}$$ 8

6. (c) Determine an initial basic feasible solution using :

(i) North-West Corner Rule

(ii) Vogel's approximation method for the following transportation problem :

	Destination					
Origin	A	B	C	D	E	Supply
I	2	11	10	3	7	4
II	1	4	7	2	1	8
III	3	9	4	8	12	9
Demand	3	3	4	5	6	

8

(d) Solve the following assignment problem :

$$\begin{bmatrix} 5 & 3 & 4 & 7 & 1 \\ 2 & 3 & 7 & 6 & 5 \\ 4 & 1 & 5 & 2 & 4 \\ 6 & 8 & 1 & 2 & 3 \\ 4 & 2 & 5 & 7 & 1 \end{bmatrix}$$ 8

UNIT—IV

7. (a) Find the expression for $P(k \geq n)$, $E(n)$ and $E(m)$ for the queueing model $\{(M|M|1) : (\infty / FIFO)\}$. 8

(b) A supermarket has two girls serving at the counters. The customers arrive in a Poisson fashion at the rate of 12 per hour. The service time for each customer is exponential with mean 6 minutes. Find :

(i) Probability that an arriving customer has to wait for service.

(ii) The average number of customers in the system.

(iii) The average time spent by a customer in the supermarket. 8

8. (c) Derive the waiting time distribution for the model $\{(M|M|1) : (\infty / FIFO)\}$ and find $E(w)$ and $E(v)$. 8

(d) At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady – state probabilities for the various number of trains in the system. Also find the average waiting time of a new train coming into the yard. 8

UNIT—V

9. (a) Show that : For any 2×2 two-person zero-sum game without any saddle point having the payoff matrix for Player A

$$\begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array}$$

the optimum mixed strategies $S_A = \begin{bmatrix} A_1 & A_2 \\ p_1 & p_2 \end{bmatrix}$ and $S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$ are determined by

$\dfrac{p_1}{p_2} = \dfrac{a_{22} - a_{21}}{a_{11} - a_{12}}, \dfrac{q_1}{q_2} = \dfrac{a_{22} - a_{12}}{a_{11} - a_{21}}$ where $p_1 + p_2 = 1$ and $q_1 + q_2 = 1$. The value V of the game to A is given by

$$V = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - (a_{12} + a_{21})}.$$ 8

(b) Solve the game graphically :

$$\begin{bmatrix} 3 & -4 \\ 2 & 5 \\ -2 & 8 \end{bmatrix}$$ 8

10. (c) Use the notation of dominance, reduce the game to a 2 × 4 game and solve it graphically :

$$\begin{array}{cc} & P_2 \\ P_1 & \begin{bmatrix} 8 & 15 & -4 & -2 \\ 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{bmatrix} \end{array}$$ 8

(d) Use linear programming to solve the game :

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix} \end{array}$$ 8