(d) Discuss the open isotropic model of the universe given by the RW metric :

$ds^2 = R^2(t)[-d\alpha^2 - \sin h^2 \alpha (d\theta^2 + \sin^2 \theta d\phi^2)] + dt^2$

with reference to dust and radiation distribution. 8

UNIT—V

9. Discuss the Schwarzschild interion solution as a simple steller model. 16

10. Derive the TOV equation

$$p = \frac{-(\rho + p/c^2)(m + 4\pi kpr^3/c^4)c^2}{r(r-2m)}$$ for

the relativistic steller structure. 16

AQ–1060

M.Sc. Part—II Semester—IV (CBCS) Examination

MATHEMATICS

(General Relativity and Cosmology—II)

Paper—XVIII-(403)

Time : Three Hours] [Maximum Marks : 80

Note :—Solve **FIVE** questions selecting one from each unit.

UNIT—I

1. (a) By considering the static homogeneous and spherically symmetric universe, obtain the Einstein's model of the universe. 8

   (b) Prove that there is no Doppler shift in the Einstein's universe. 8

2. (c) Prove that the geometry of the de-Sitter universe is that of the surface of a sphere embedded in five dimensional Euclidean space. 8

   (d) Discuss the velocity and acceleration of a particle in the de-Sitter universe and show that the particle at rest at origin will remain permanently at rest at the origin. 8

UNIT—II

3. (a) State and explain :

Cosmological principle. 4

(b) By making the necessary assumptions, derive the Robertson-Walker line element. 12

4. (c) Prove that the quantity $\frac{1}{a^2}$ in the line element

$$ds^2 = -e^{g(t)}\left[\frac{dr^2}{1-r^2/a^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\theta^2\right] + dt^2$$ is

the Riemannian curvature of the spatial extent at any given time t. 8

(d) Considering the spatial part of the closed form of RW model, Prove that the total proper spatial volume at any given time t is $V = 2\pi^2 R^3$. 8

UNIT—III

5. (a) Discuss the motion of particle in R- W model and prove that a material particle does not experience the gravitational force. 8

(b) Prove that the equation of light trajectory in the RW universe is given by

$$r = za \tan\left(\frac{e^{-bt_1} - e^{-bt_2}}{2ab}\right).$$ 8

6. (c) Prove that in RW model, there exists Red shift for expanding universe and a blue shift for contracting universe. 8

(d) Prove that the equation for red shift z in terms of ℓ is :

$$z = R_o' \ell + \frac{R_o'^2}{2}(1+q_o)\ell^2 + O(\ell^3),$$

where q_o is the deceleration parameter. 8

UNIT—IV

7. (a) Obtain the fundamental equation of dynamic cosmology :

$$\dot{R}^2 + k = \frac{8\pi\rho R^2}{3}.$$ 8

(b) Prove that the matter dominated era of the universe is governed by the equation :

$$\left(\frac{\dot{R}}{R_o}\right)^2 = H_o^2\left\{1 - 2q_o + 2q_o \cdot \frac{R_o}{R}\right\},$$

where the symbols have their convensional meanings. 8

8. (c) Derive the equation for the age of the universe to, for Friedmann closed model in the form

$$t_o = \frac{q_o}{H_o (2q_o^{-1})^{3/2}}\left\{\cos^{-1}\left(\frac{1}{q_o} - 1\right) - \frac{1}{q_o}\left(2q_o - 1\right)^{1/2}\right\}.$$ 8