UNIT-V

9. (a) Define potential function and find the condition that any one parameter family of surface $f(x, y, z) = c$ form a family of equipotential surface. 8

(b) Show that the solution of $u(x, t)$ of P. D. E. $u_t - ku_{xx} = f(x, t)$, $0 < x < t$, $t > 0$.

subject to condition,

$$u(x, 0) = f(x), \; 0 \le x \le \ell$$

$$u(0, t) = u(\ell, t) = 0, \; t \ge 0,$$

is unique. 8

10. (c) Solve the partial differential equation :

$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ subject to the condition, $0 < x < \ell$, $t > 0$,

$u(0, t) = u(\ell, t) = 0, \; t \ge 0$

and $u(x, 0) = f(x)$; $0 \le x \le \ell$. 8

(d) If $\Phi = \Phi(\gamma, \theta, \psi)$ is harmonic function, then prove that $\overline{\Phi} = \frac{a^2}{r}\Phi\left(\frac{a^2}{r}, \theta, \Psi\right)$ is also a harmonic function, where, (γ, θ, ψ) are the spherical polar co-ordinates and 'a' is a constant. 8

AQ–1059

M.Sc. (Part-II) Semester–IV (CBCS) Examination

MATHEMATICS–402

(Partial Differential Equations)

Time—Three Hours] [Maximum Marks—80

Note :— Solve **one** question from each unit.

UNIT- I

1. (a) Prove that necessary and sufficient condition for integrability is $[F, g] = 0$. 8

(b) Find the complete integral of the equation $p^2x + q^2y = z$, by Jacobi's method. 8

2. (c) Explain the following terms :

(i) Complete Integral.

(ii) General Integral.

(iii) Singular Integral. 8

(d) Find the integral surface of the differential equation $(x - y) p + (y - x - z) q = z$, passing through the circle $z = 1$, $x^2 + y^2 = 1$. 8

UNIT-II

3. (a) Prove that every integral surface is generated by family of characteristic curves. 8

(b) Reduce the equation $u_{xx} - x^2u_{yy} = 0$ to a canonical form. 8

4. (c) Solve the Cauchy problem for $2z_x + yz_y = z$ for the initial data curve $\in$: $x_o = S$, $y_o = S^2$, $z_o = s$, $1 \le s \le 2$. 8

(d) Reduce the equation $u_{xx} + 2xu_{xy} + x^2u_{yy} = 0$ to a canonical forms and solve wherever possible. 8

UNIT-III

5. (a) Find the solution of the vibration of an infinite string. The initial conditions are :

$$y(x, 0) = f(x)$$

$$yt(x, 0) = g(x)$$

$$-\infty < x < \infty ;$$

where, f(x) is initial position of the string and g(x) is the initial velocity at point x. 8

(b) Prove that for the equation :

$$L_u = U_{xy} + \frac{1}{4}u = 0$$, the Riemann function is

$$v(x, y, \alpha, \beta) = Ju\left(\sqrt{(x-\alpha)(y-\beta)}\right).$$ 8

6. (c) State and prove Green's theorem. 8

(d) Solve $y_{tt} - c^2 y_{xx} = 0 \quad 0 < x < 1, t > 0$

$y(0, t) = y(1, t) = 0$ and

$y(x, 0) = x(1 - x), 0 \le x \le 1.$

$yt(x, 0) = 0, \quad 0 \le x \le 1.$ 8

UNIT–IV

7. (a) Find the solution of the problem :

$$\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0, r < a$$, subject to the boundary condition $u(a, \theta) = f(\theta)$. 8

(b) Let u (x, y) be harmonic in a bounded domain D and continuous in $\overline{D} = D \cup B$. Then prove that u attains its maximum on the boundary B of D. 8

8. (c) State and prove uniqueness theorem for Laplace equation. 8

(d) Find the solution of the problem

$$\nabla^2 u = u_{xx} + u_{yy} = 0, \quad 0 < x < a, 0 < y < b$$

with boundary condition given below,

$u(x, 0) = f(x), \ 0 \le x \le a,$

$u(x, b) = 0 \quad , 0 \le x \le a,$

$u(0, y) = 0 \quad , 0 \le y \le b,$

$u(a, y) = 0 \quad , 0 \le y \le b.$ 8

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